"Build a Lorenz Attractor"

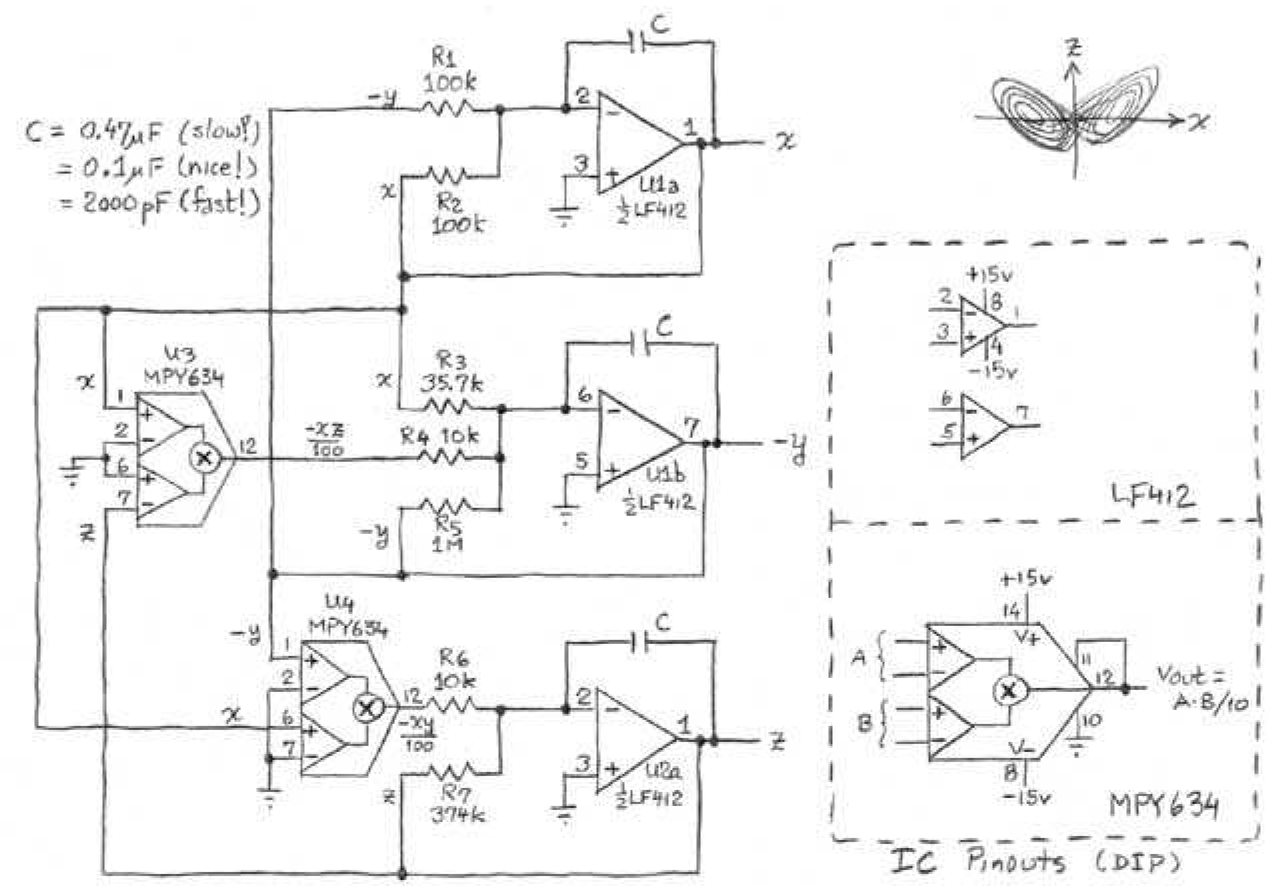
In 1963 Edward Lorenz published his famous set of coupled nonlinear first-order ordinary differential equations; they are relatively simple, but the resulting behavior is wonderfully complex. The equations are:

dx/dt = s(y-x)
dy/dt = rx-y-xz
dz/dt = xy - bz

with suggested parameters s=10, r=28, and b=8/3. The solution executes a trajectory, plotted in three dimensions, that winds around and around, neither predictable nor random, occupying a region known as its *attractor*. With lots of computing power you can approximate the equations numerically, and many handsome plots can be found on the web. However, it's rather easy to implement these equations in an analog electronic circuit, with just 3 op-amps (each does both an integration and a sum) and two analog multipliers (to form the products xy and xz).

The Circuit

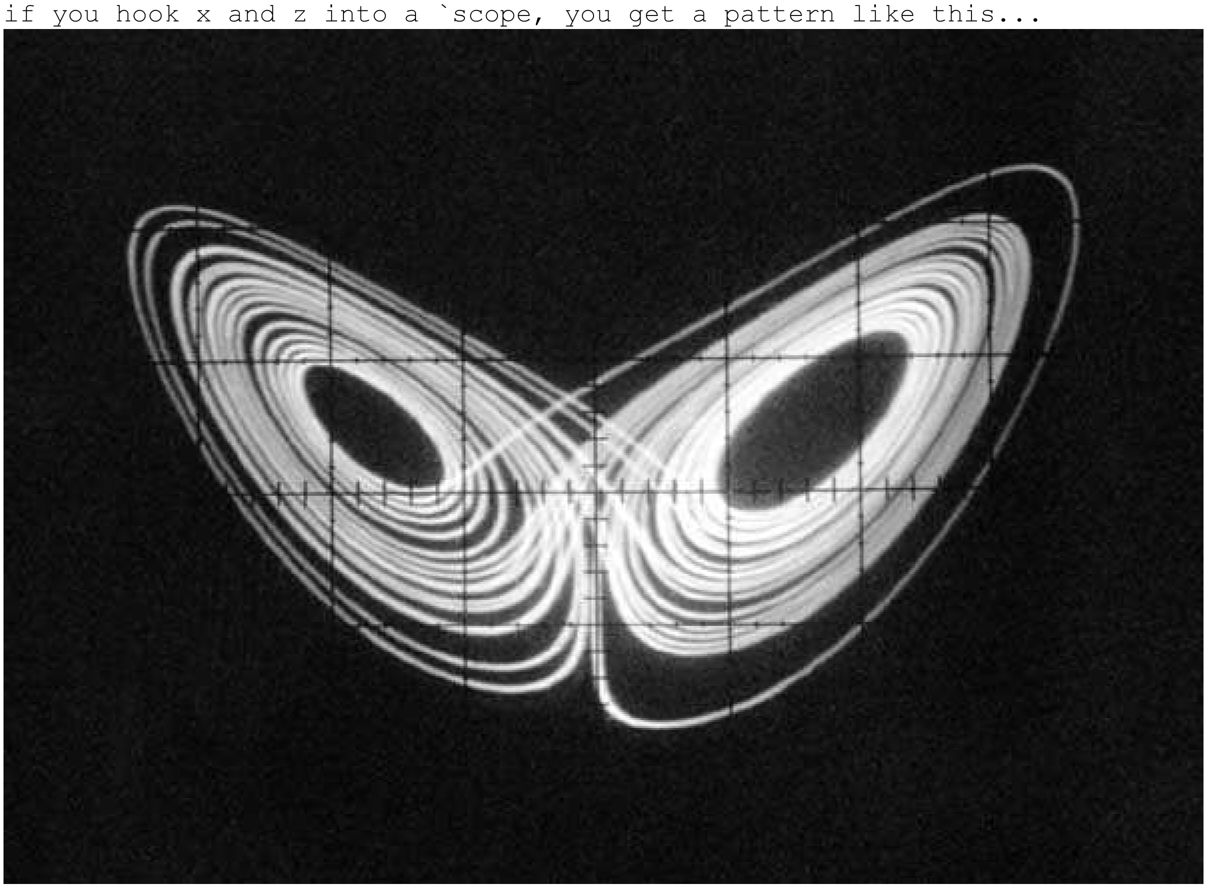
Here's the circuit:



It's not hard to understand: the op-amps are wired as integrators, with the various terms that make up each derivative summed at the inputs. The resistor values are scaled to 1 megohm, thus for example R3 weights the variable x with a factor of 28 (1M/35.7k); this is combined with -y and -xz, each with unit weight. (note: the equations on the diagram are normalized to 0.1V, hence the multiplier scale factor of 100.)

The Output

The circuit just sits there and produces three voltages x(t), y(t), and z(t); if you hook x and z into a `scope, you get a pattern like this...



...the characteristic "owl's face" of the Lorenz attractor. The curve plays out in time, sometimes appearing to hesitate as it scales the boundary and decides which basin to drop back into. The value of C, the three integrator capacitors, sets the time scale: at 0.47uF it does a leisurely wander; at 0.1uF it winds around like someone on a mission; and at 0.002uF it is fiercely busy solving its equations and delighting its audience.