

## Chua's Circuit for High School Students

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*Abstract:* A step-by-step procedure to build Chua's circuit using easily available and inexpensive components is presented. We illustrate the classic breadboard method for building Chua's circuit. For readers without access to an oscilloscope, this paper proposes the use of a laptop/Personal Computer soundcard to capture the voltage waveforms generated from the circuit. The reader can then plot the waveforms on a computer screen using the software provided by the authors.

### 1. Introduction

Have you ever noticed water dripping from a tap, which is sometimes regular and sometimes irregular? Or water flowing through an obstacle in a way that can be either smooth (laminar) or turbulent?

At some point in time we all notice these phenomenons, however they are usually very hard to explain. It was only after Edward Lorenz [Lorenz, 1963] came to conclude that his computer simulated weather model is highly sensitive to initial conditions that the scientific community rigorously studied the phenomenon of chaos. However it took more than 30 years to prove that the observations made by Lorenz were indeed chaotic [Tucker, 2000].

Chaos, along with Quantum Mechanics and Relativity, has been hailed as one of the major discoveries of 20<sup>th</sup> century. Chua's circuit was the first circuit implementation specially designed to exhibit chaos [Chua, 1984] and the first that was proved to be chaotic rigorously [Chua, et. al. 1986]. Chua's circuit is also the *simplest* [Chua et. al., 1986] physical system where chaos can be observed.

Here we show how easily high school students can build this simple inexpensive electronic circuit in less than an hour for approximately \$10. The organization of this paper is: first we give a simple non-mathematical introduction to Chua's circuit. This is followed by the crux of the paper – building Chua's circuit. This second section is further sub-divided into a component check-list section, a breadboard primer and building the actual circuit. The third section shows how to use the PC's sound card as an oscilloscope. The fourth section in this paper talks about a simple Chua's circuit kit that you can order for building Chua's circuit in 5 minutes! This is followed by a FAQ section. We conclude the paper with a references section.

The aim of this paper is that high school students can build Chua's circuit *quickly* and *easily*. Therefore, we have *explicitly* avoided the mathematical intricacies of Chua's

circuit. We encourage the reader to consult the references for the mathematical details of Chua's circuit and chaos.

## 2. Introduction to Chua's circuit

A schematic of Chua's circuit is shown in figure 1. The circuit consists of three energy storing elements (two capacitors  $C_1$  and  $C_2$ ; one inductor,  $L$ ), one linear resistor,  $R$  and one non-linear resistor described by the current versus voltage characteristics  $i_R=f(v_R)$  as shown in Figure 2.

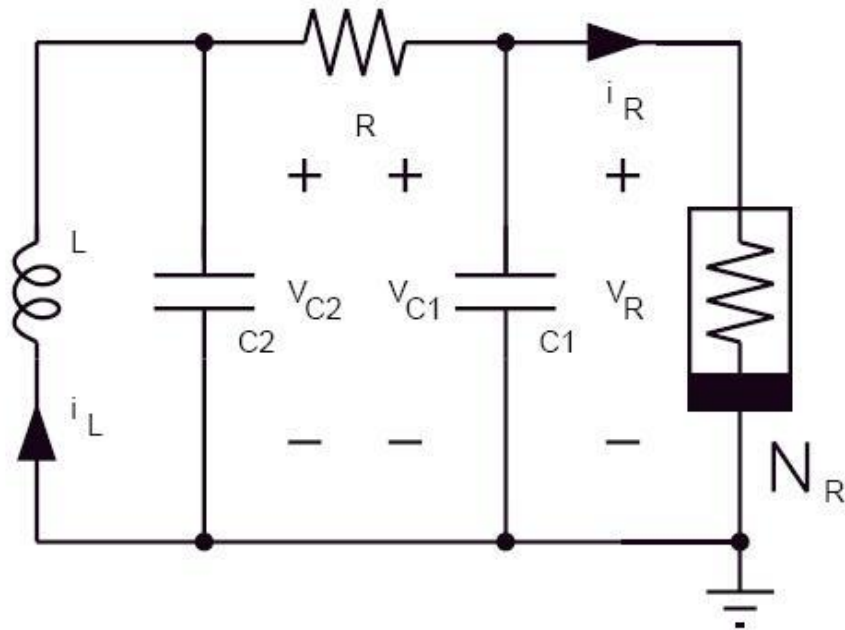


Figure 1. Chua's Circuit

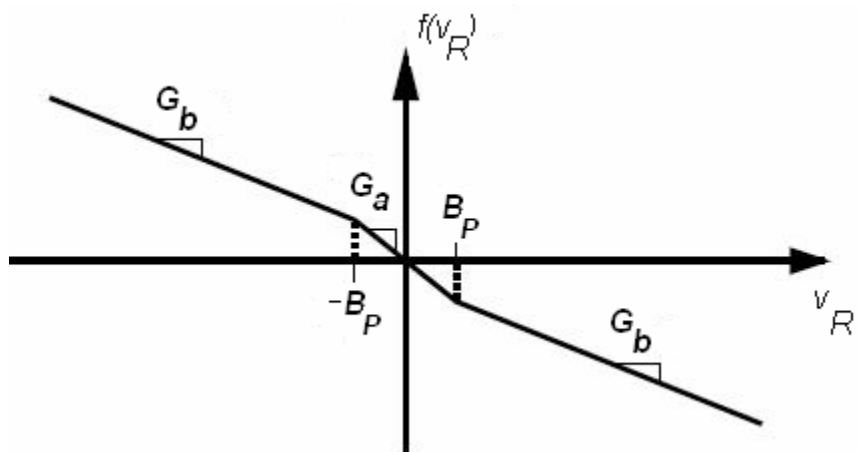


Figure 2. Current vs. voltage characteristics of the Chua Diode

### 3. Building Chua's Circuit

#### a. Components required

Component	Quantity	Tolerance
10 nF Capacitor	1	5%
100 nF Capacitor	1	5%
18 mH Inductor	1	5%
220 $\Omega$ resistor	2	5%
2.2k $\Omega$ resistor	1	5%
22k $\Omega$ resistor	1	5%
3.3k $\Omega$ resistor	1	5%
2k Potentiometer	3	Not applicable
9 V Batteries	2	Not applicable
Breadboard	1	Not applicable
LMC6482 op-amp	2	Not applicable
Connecting Wires	1	Not applicable
Audio plug	1	Not applicable

**Table 1.** A list of components for building Chua's circuit

■ The inductor can be obtained from Coilcraft: <http://www.coilcraft.com> (part number: PCH-45-186L).

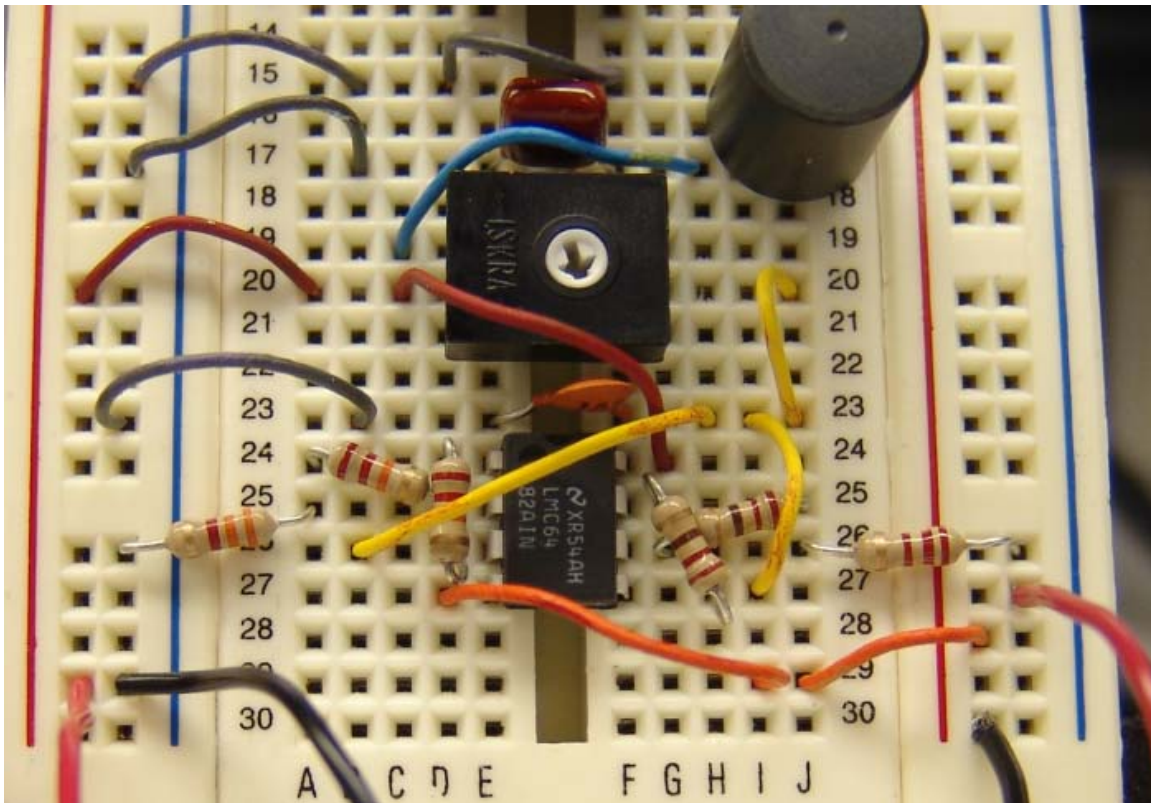
■ The other components can be obtained from your local RadioShack store or ordered via DigiKey (<http://www.digikey.com>). The part number for the audio plug is SC1127-ND.

■ You can download the software for turning your PC's soundcard into an oscilloscope at: [www.chuacircuit.com](http://www.chuacircuit.com)

1. Identify the different components and write their values on them. You can consult Appendix I for identifying different component values.
2. With the help of Appendix II, mark the pin configuration for the LMC6482 operational amplifier. You can also use other op-amps like the TL082 or the AD712.

#### b. A breadboard primer

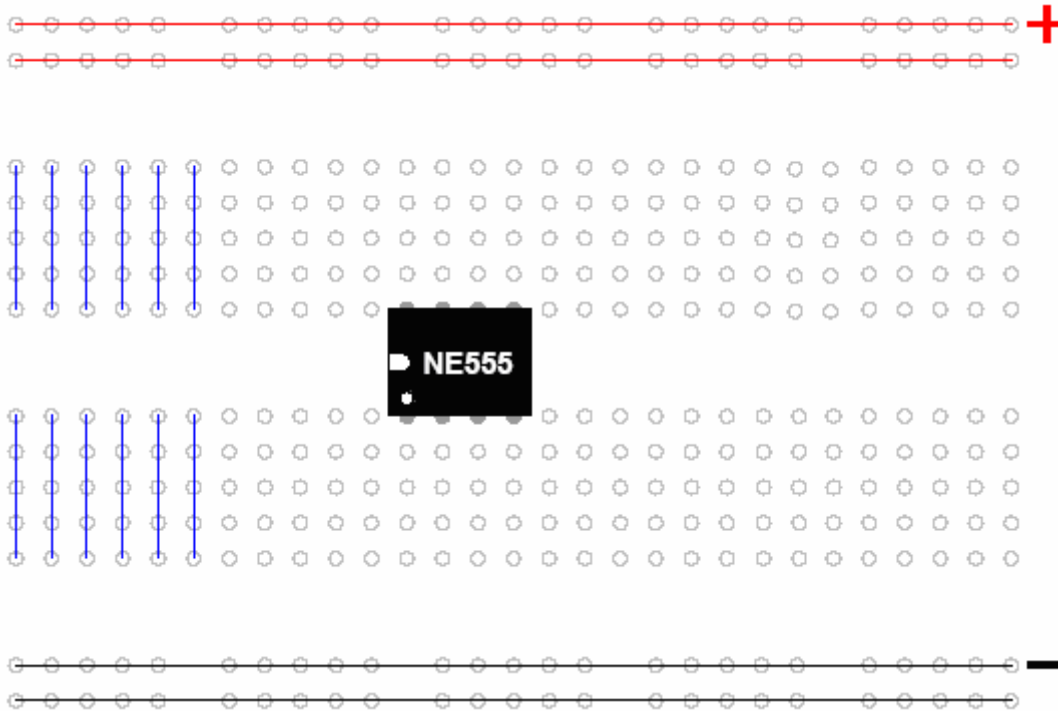
A breadboard is a reusable device for building an electronic circuit [Breadboard Wiki, 2007]. Figure 3 shows Chua's circuit on a breadboard.



**Figure 3.** Chua's circuit as seen on a breadboard. Audio plug for computer interface not shown.

Components are easy to change or replace on a breadboard. Parts will not be damaged so they will be available for re-use afterwards.

Breadboards have many tiny sockets (called 'holes') arranged on a 0.1" grid. The leads of most components can be pushed straight into the holes. Integrated circuits (ICs) are inserted across the central gap with their notch or dot to the left. Figure 4 shows how the breadboard holes are connected. The top and bottom rows are linked **horizontally** all the way across. The power supply is connected to these rows, + at the top and 0V (zero volts) at the bottom. We suggest using the upper row of the bottom pair for 0V, then you can use the lower row for the negative supply with circuits requiring a dual supply (e.g. +9V, 0V, -9V). The other holes are linked **vertically** in blocks of 5 with no link across the centre.



**Figure 4.** Breadboard connections illustrated, along with an IC (NE555) [Electronics Club, 2007]

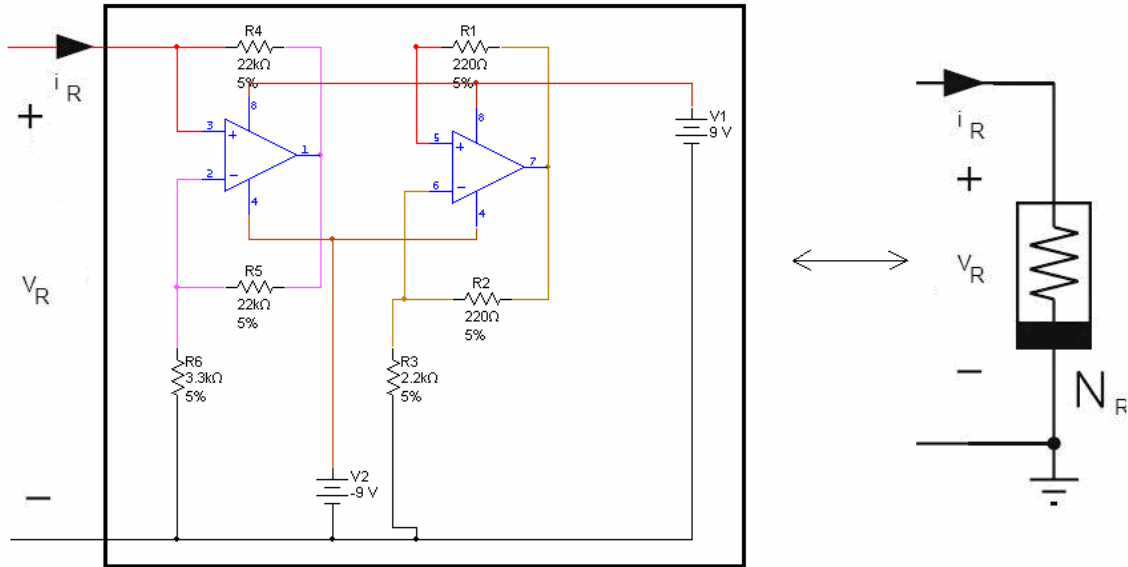
### c. The breadboard version of Chua's circuit: A Step-by-Step Guide

Converting the Chua circuit diagram to a breadboard layout is straightforward provided that you follow the simple rules below:

1. When putting parts on a breadboard you must concentrate on their **connections**, not their positions on the circuit diagram.
2. Start the wiring with the ICs first. Place it in the centre of the breadboard and work round it pin by pin, putting in all the connections and components for each pin in turn.

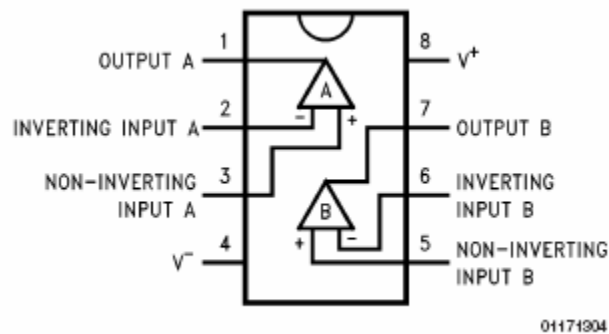
We will follow the two rules above to connect Chua's circuit. Probably the most difficult part in wiring up Chua's circuit is the nonlinear element (Chua diode) so we will start with that first.

1. Figure 5 shows how we can realize the current vs. voltage characteristic from figure 2. We are using the two op-amp implementation of the Chua diode from Kennedy [Kennedy, 1992].



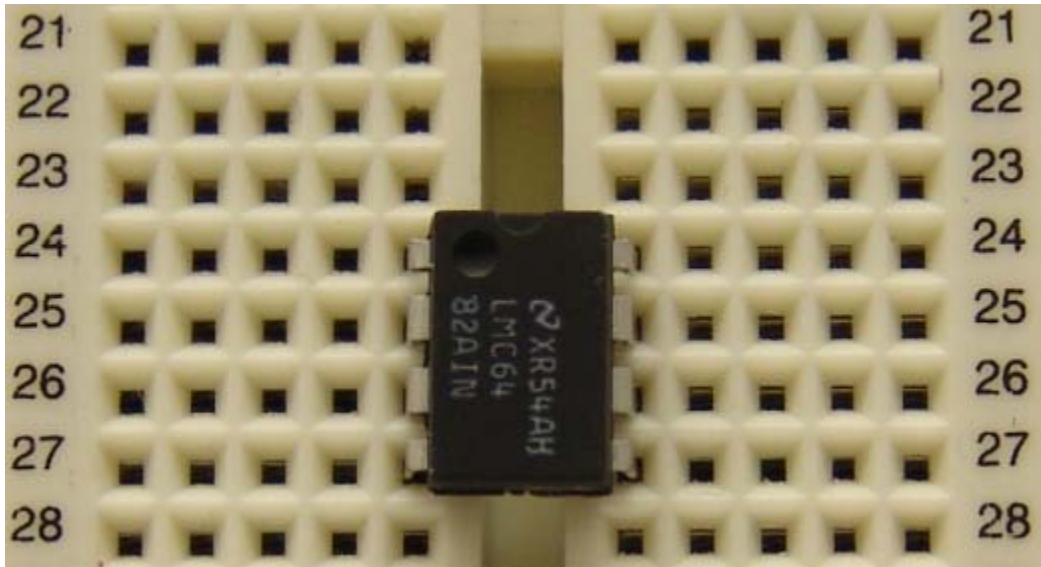
**Figure 5.** Chua diode realized using M.P. Kennedy's two op-amp realization

The Chua diode is realized using operational amplifiers (op-amps). Appendix II gives a brief introduction to op-amps. From Appendix II, you can easily infer the connection diagram (pin-out) of the op-amp, this is the most important piece of information you need for connecting the op-amp. The pin-out is repeated below in figure 6 for convenience. The op-amp that we will be using is the LMC6482.



**Figure 6.** LMC6482 op-amp pinout

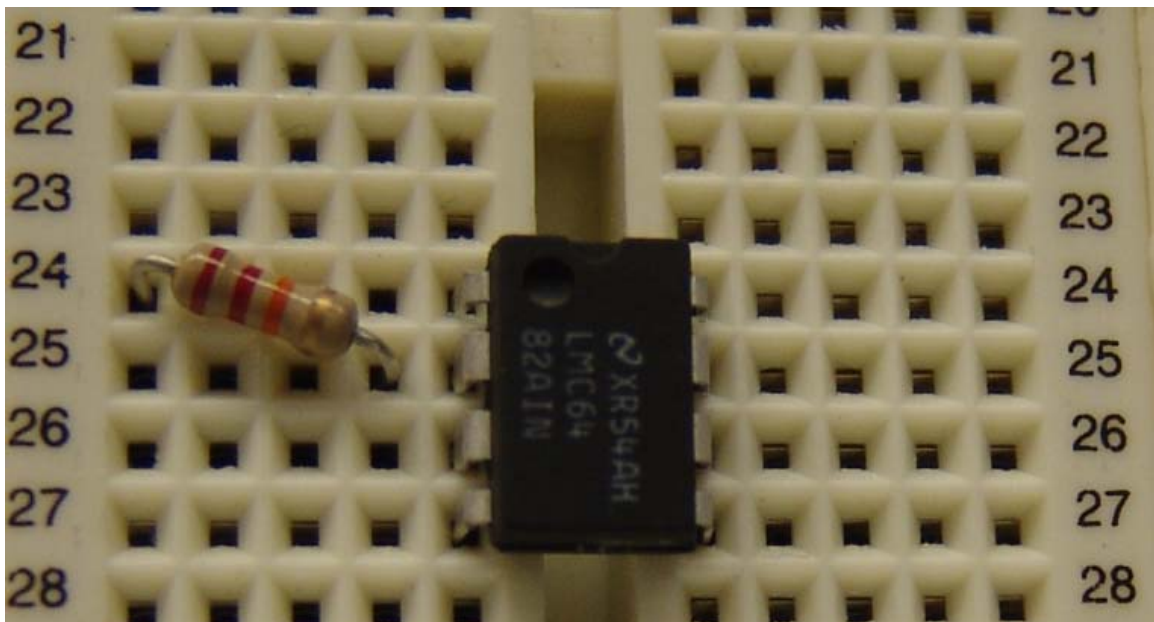
2. Notice the *triangle circuit symbol* actually represents the op-amp. However, using figures 5 and 6 you can easily wire up the op-amp circuit for the Chua diode. Before you proceed make sure that you have sorted out your resistors and capacitors, refer to Appendix I. For wiring up the Chua diode, you will need two 22k resistors, two 220 ohm resistors, one 3.3k and one 2.2k resistor. Lets go through the wiring step-by-step:
3. First, insert the op-amp across the middle of the breadboard, refer to figure 7.



**Figure 7.** Op-amp inserted on my breadboard.

As shown in figure 7, breadboards usually have column numbers on the sides so circuit wiring is easy. I will use the column numbers shown in figure 7 (number 21 through 28).

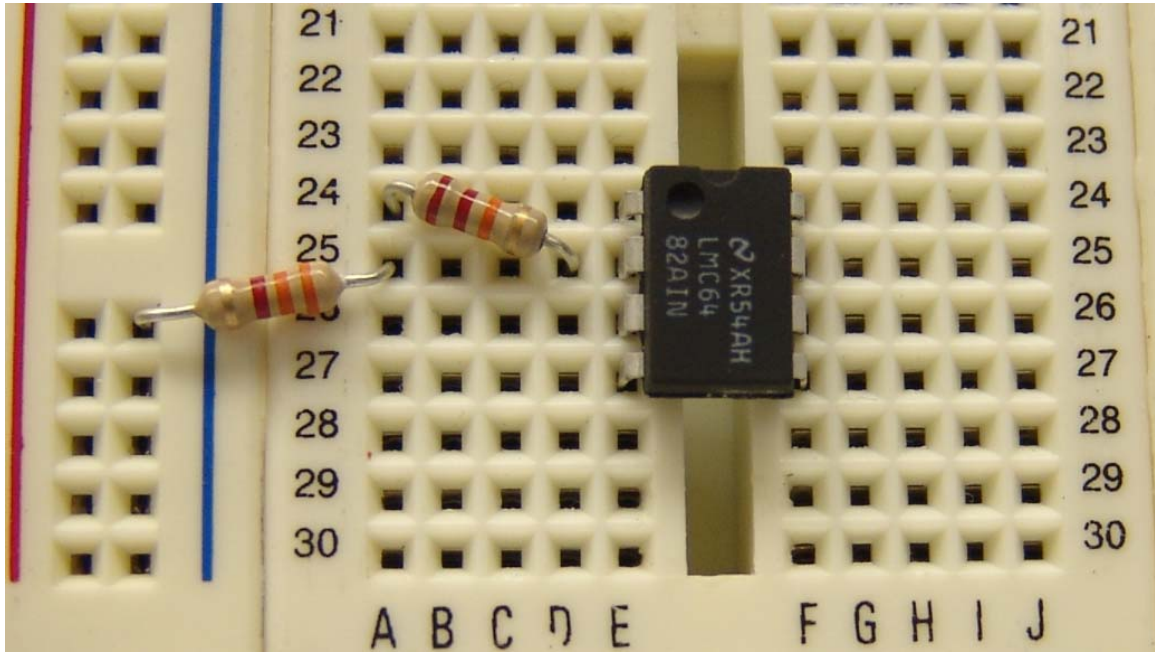
4. Insert resistor R5 (22k) from figure 5 across pins 1 and 2 of the operational amplifier. On the breadboard, pin 1 is on line 24 and pin 2 on line 25. Refer to figure 8.



**Figure 8.** Inserting the 22k resistor across pins 1 and 2 of the operational amplifier

5. Insert resistor R6 (3.3k) from figure 5 from pin 2 of the operational amplifier to the negative terminal of the battery. On the breadboard, pin 2 is on line 25.

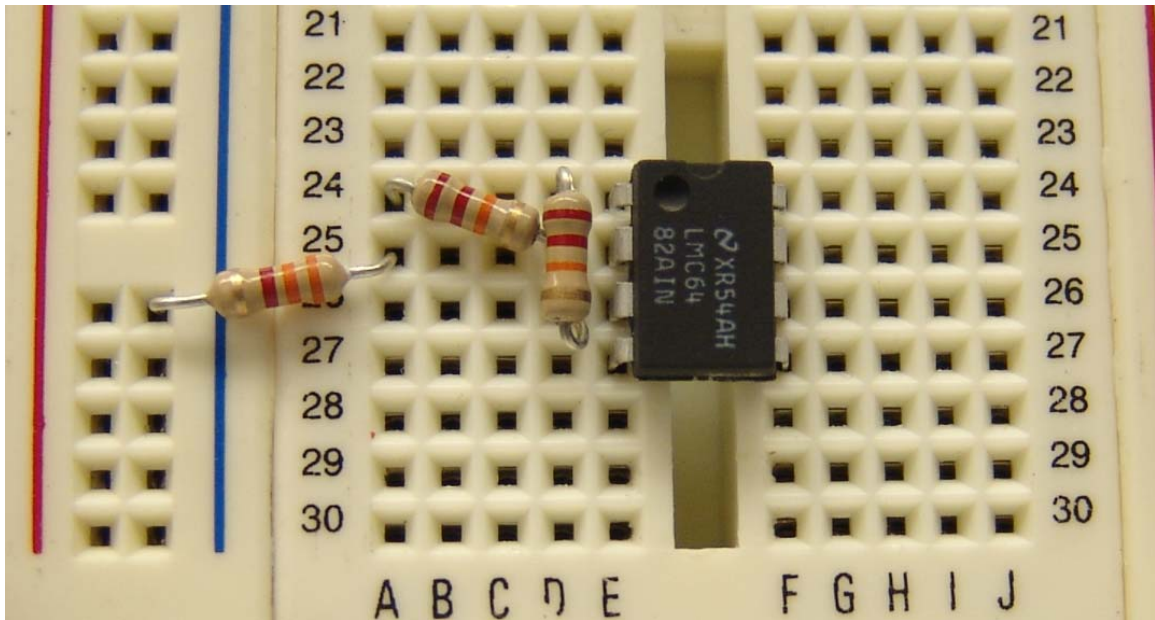
Notice that I have decided to use the vertical strips on the sides of my breadboard for positive (power) and negative (ground) terminals of my battery (refer to the breadboard primer section for more information). Refer to figure 9, the breadboard socket next to the blue strip will be the negative terminal of the battery.



**Figure 9.** Inserting the 3.3k resistor from pin 2 to ground

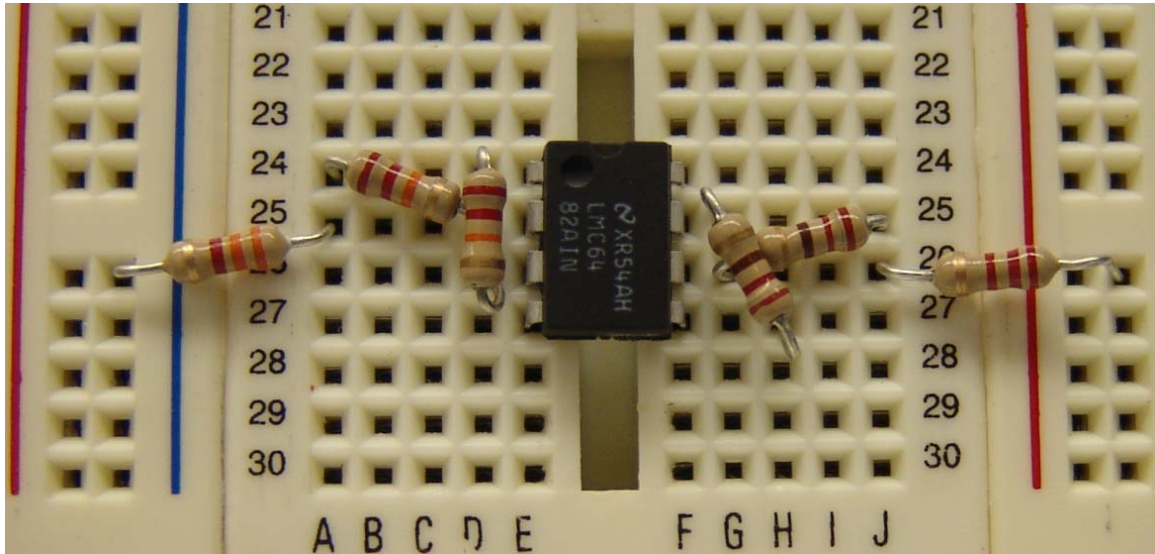
6. Insert resistor R4 (22k) from figure 5 from pin 1 of the operational amplifier to pin 3 of the operational amplifier. On the breadboard, pin 1 is on line 24 and pin 3 on line 26. Refer to figure 10.





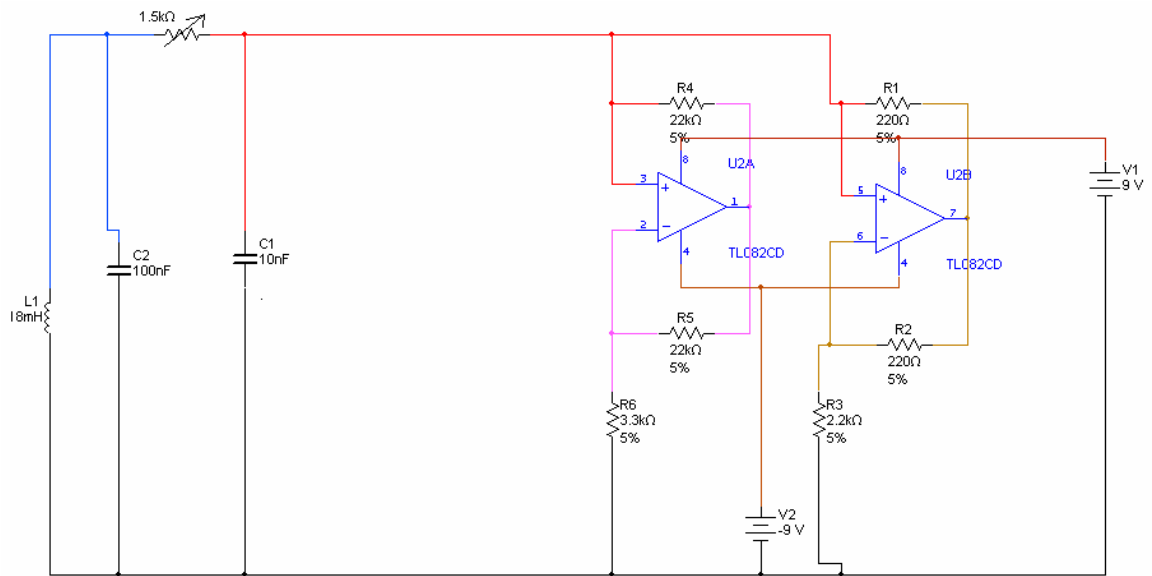
**Figure 10.** Inserting the 22k resistor across pins 1 and 3 of the operational amplifier

7. This completes one op-amp connection in figure 5. Now, complete the second op-amp connection using figure 5 as a reference. Figure 11 shows the final result for the Chua diode.



**Figure 11.** Completed Chua's diode

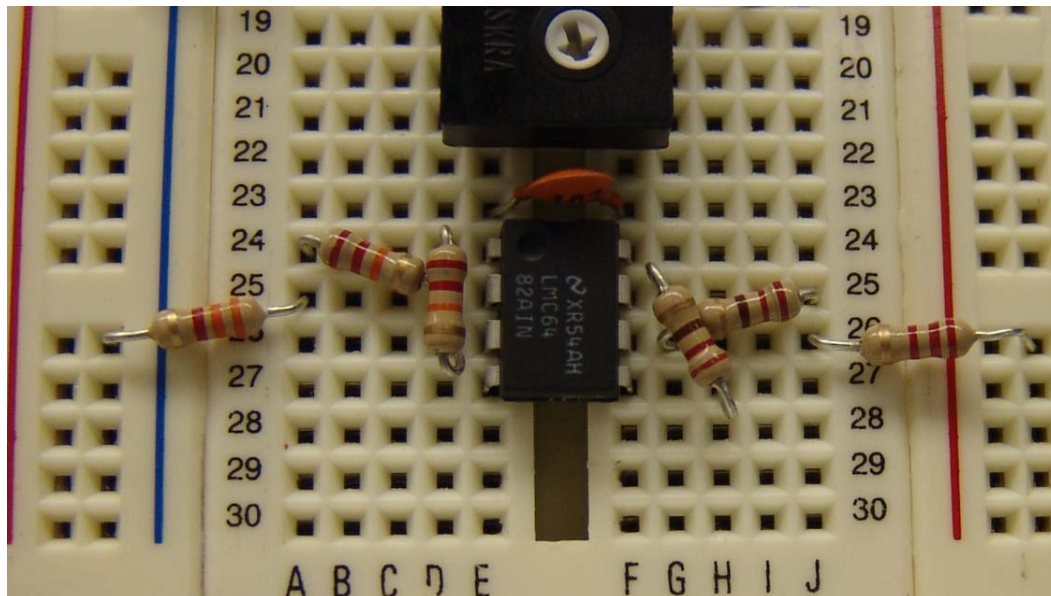
8. Figure 12 shows Chua's circuit, with the Chua diode shown explicitly. We will now put in the capacitors, potentiometer and inductor. Next we will finish the wiring and connect the batteries.



**Figure 12.** The version of Chua's circuit that we are building

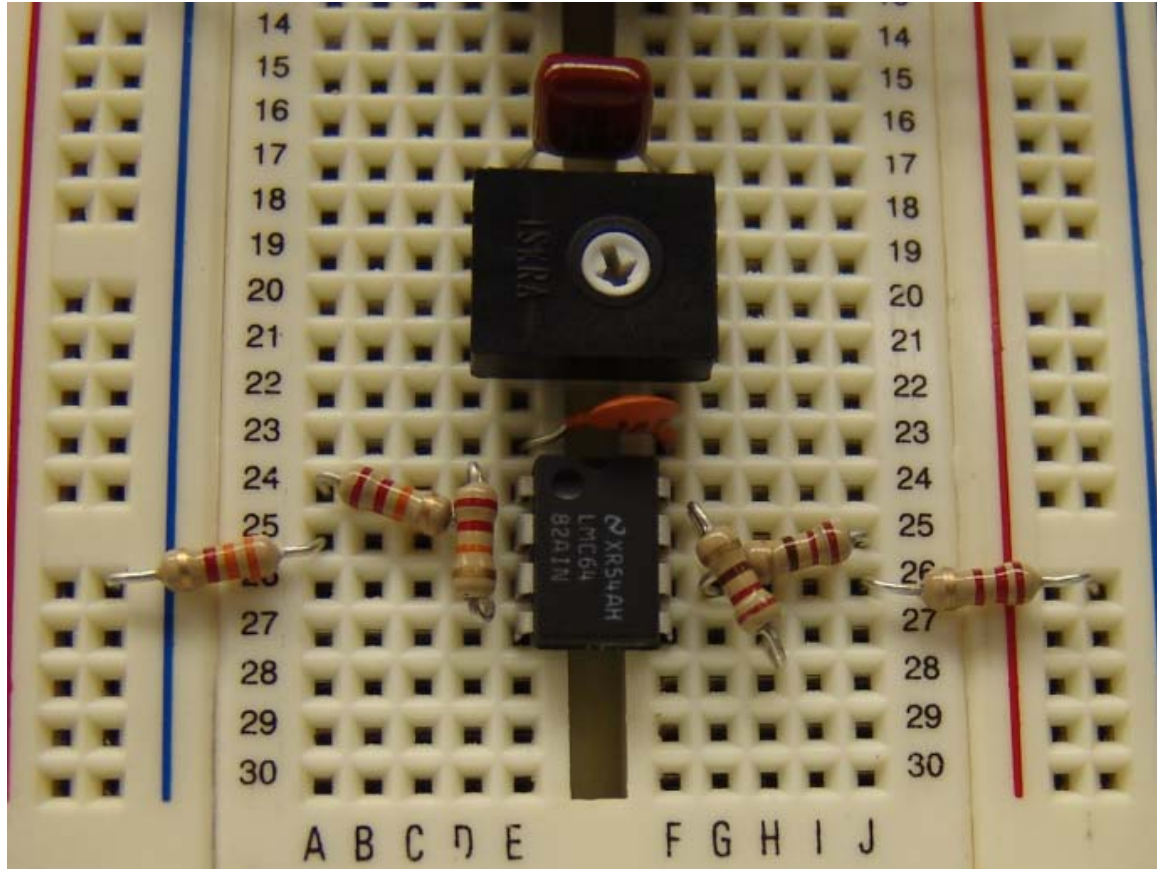
I will now start using the column labels on the breadboard for clarity. On my breadboard, the columns are indicated by the letters A through J. So I will refer to a connection point on the breadboard as D24. Notice D24 refers to the line where pin 1 of the op-amp is connected.

9. Insert the 10 nF capacitor across E23 and F23. Also insert the potentiometer on the breadboard. Refer to figure 13. Notice the potentiometer has three legs, make sure all three are inserted properly. On my breadboard, the legs are at A19, A21 and G20.



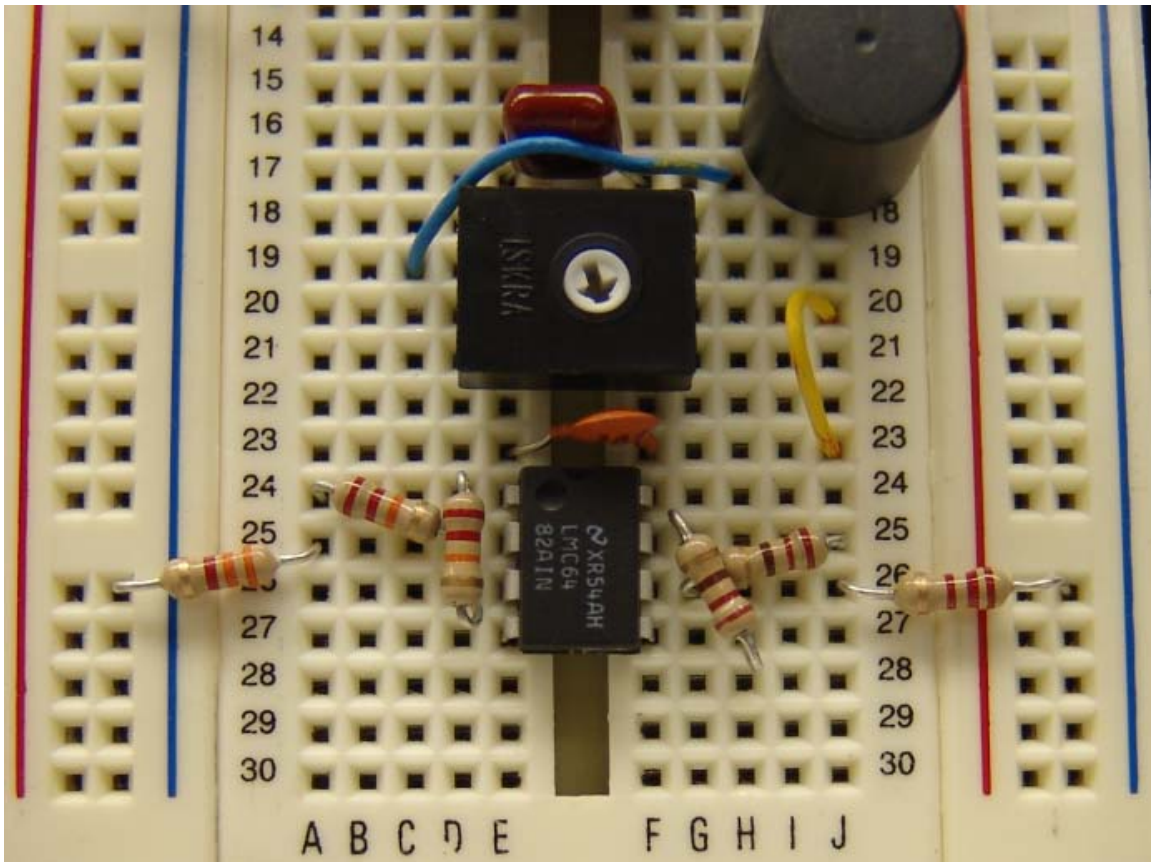
**Figure 13.** Capacitor C1 (10 nF) has been inserted across E23 and F23. Also shown is the potentiometer.

10. Next, insert the 100 nF capacitor across E17 and F17. Refer to figure 14.



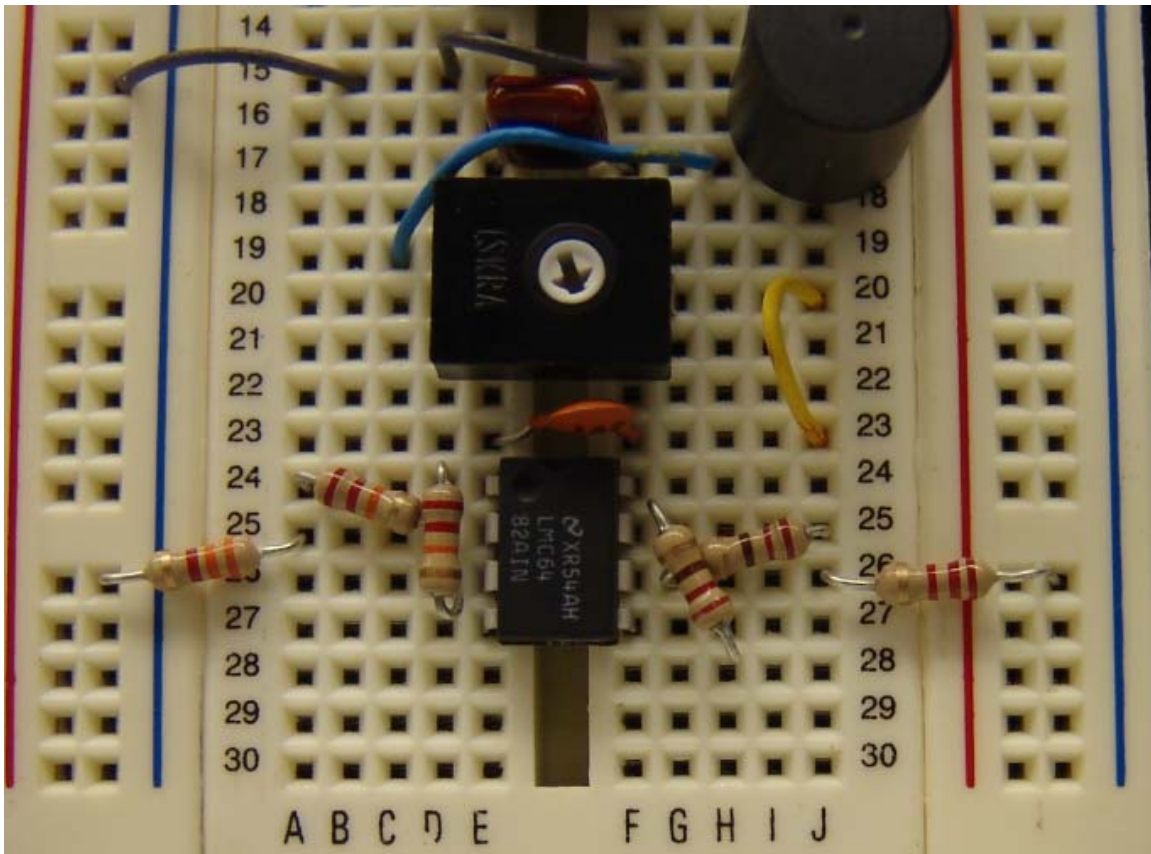
**Figure 14.** Capacitor C2 (100 nF) inserted across E17 and F17.

11. Insert the inductor across J17 and J16. We will also start connecting the two capacitors to the two ends of the potentiometer. Refer to figure 15.



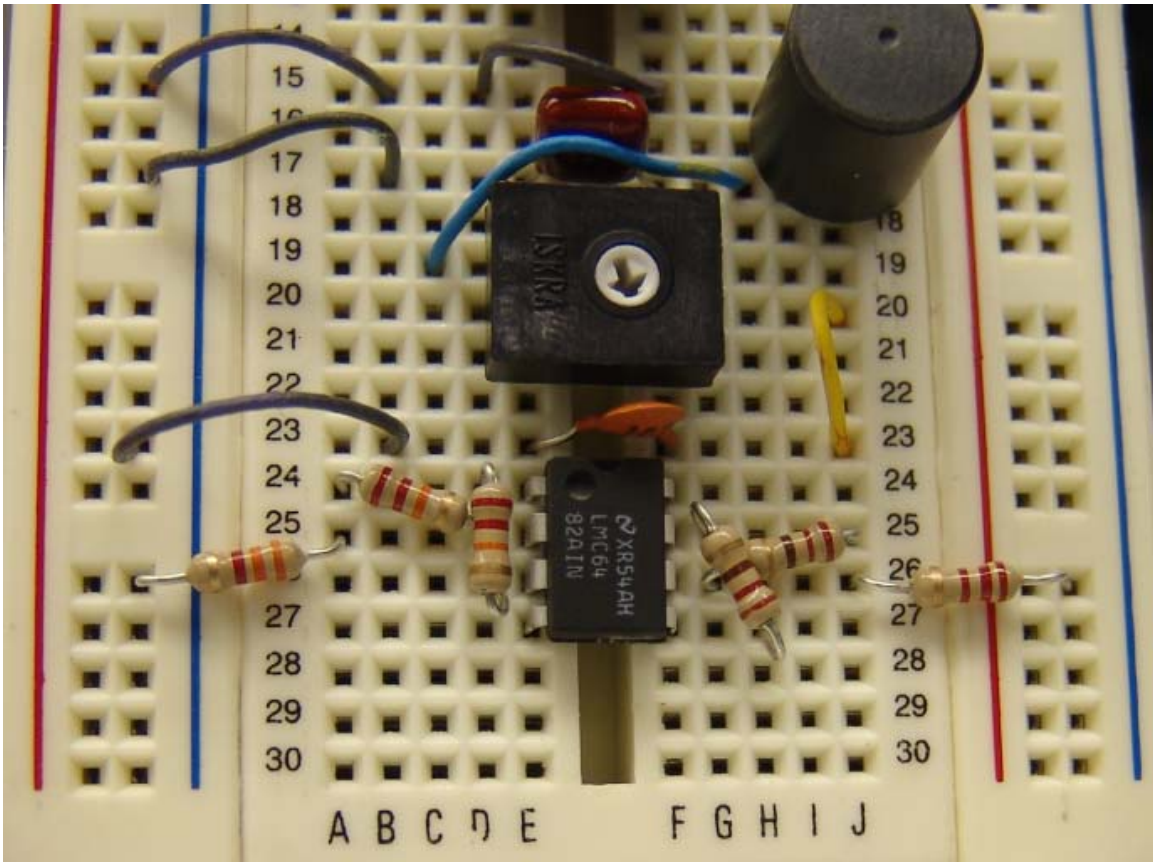
**Figure 15.** Inductor is inserted (across J17 and J15), the two capacitors have been connected to two ends of the potentiometer (the blue wire and the yellow wire)

12. In figure 16, we have wired one end of the inductor to the common negative terminal of the battery.



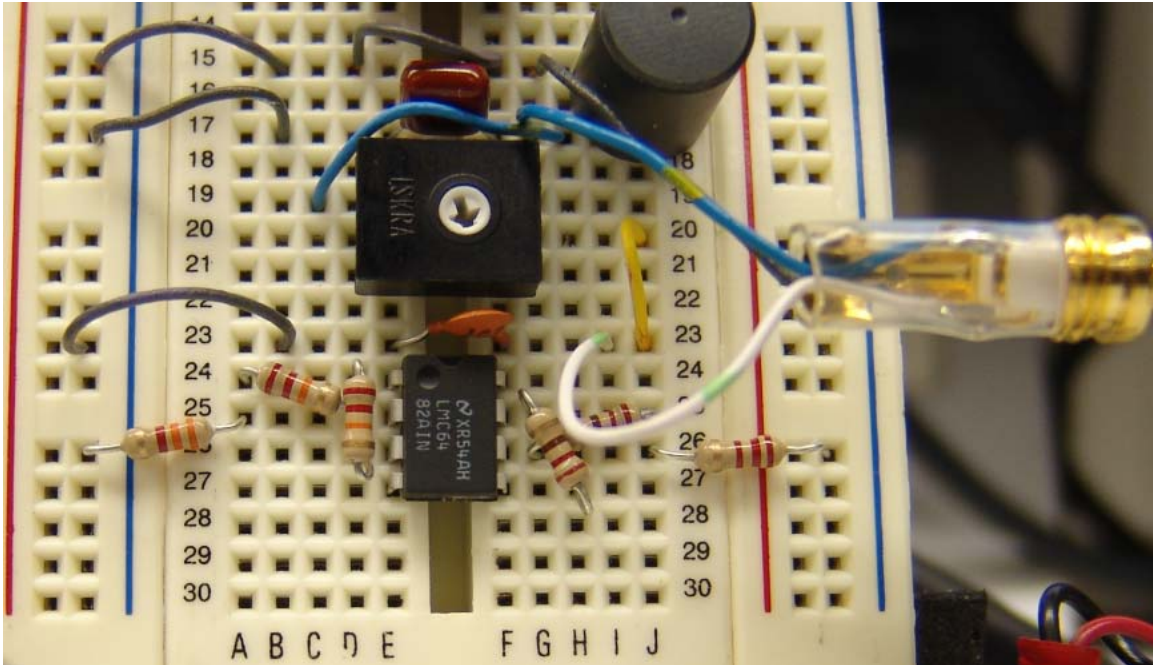
**Figure 16.** Wire one end of the inductor to the negative battery terminal. I use F15 to D15, then B15 to the negative battery terminal.

13. Connect the ends of the capacitor to the common battery terminal, refer to figure 17.



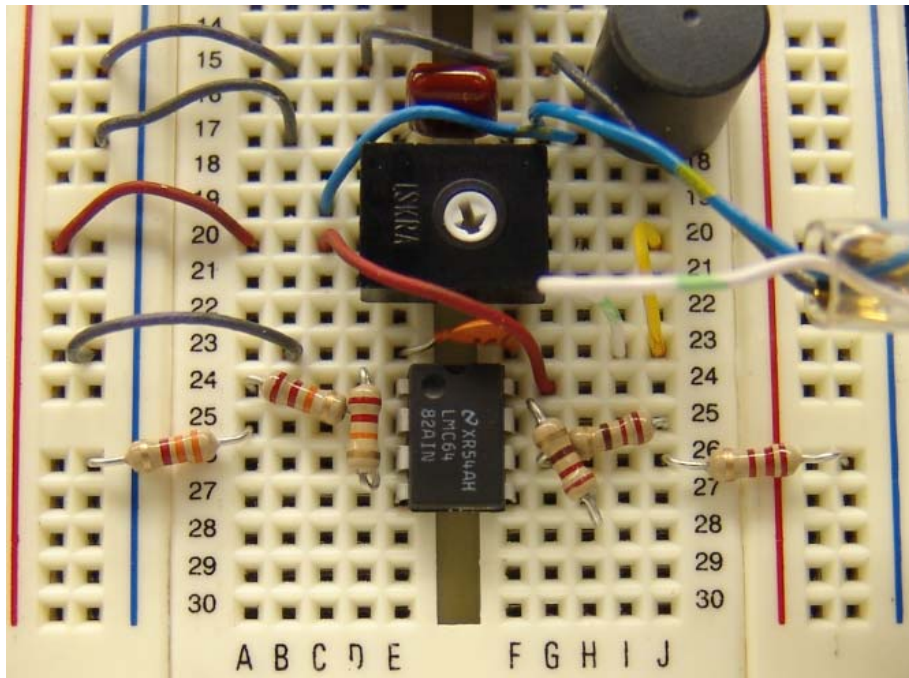
**Figure 17.** Connect B17 and B23 to the negative battery terminal

14. Now, unscrew the audio plug housing. This should reveal three wires, refer to figure 18 for the correct connection. The middle wire goes to the negative terminal of the battery, the other two go to the capacitor.



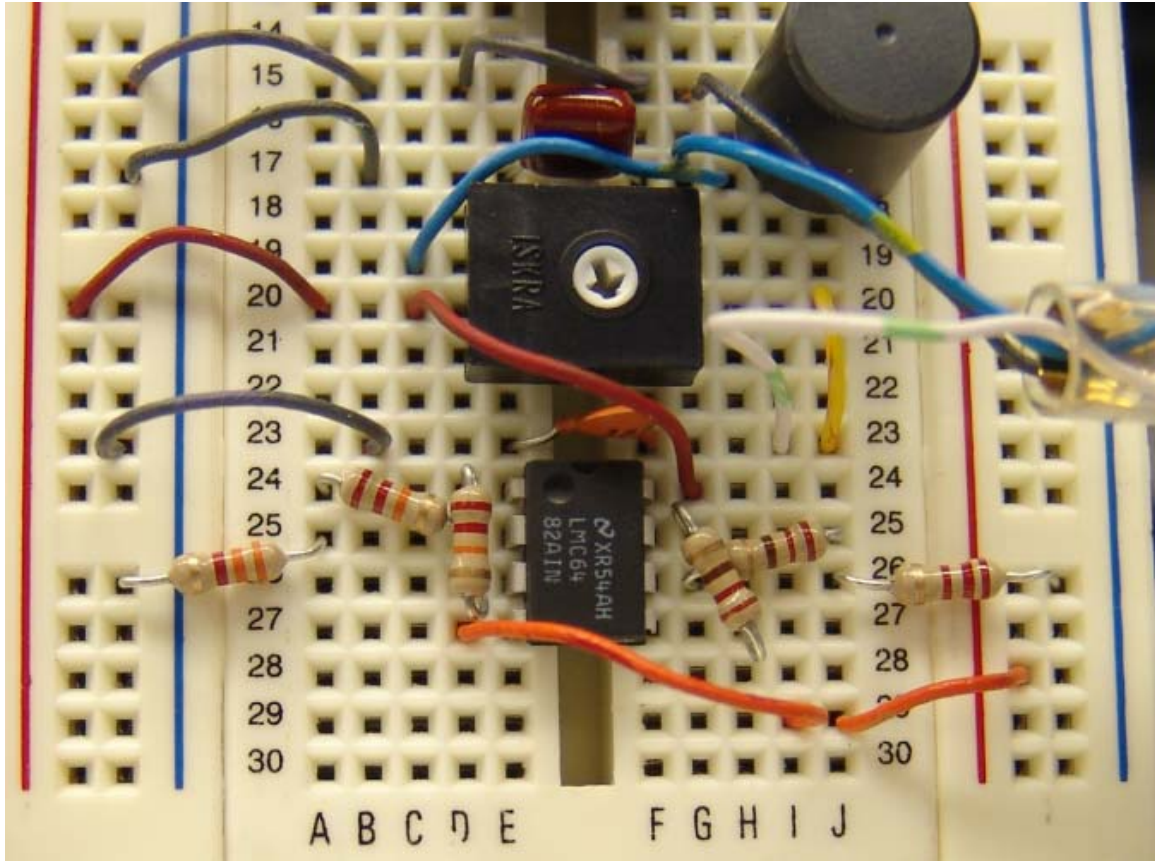
**Figure 18.** Connect the terminals of the audio plug. The middle terminal goes to G15, the other two go to G17 and I23.

15. Connect the positive power supply terminal of the op-amp. Refer to figure 19.



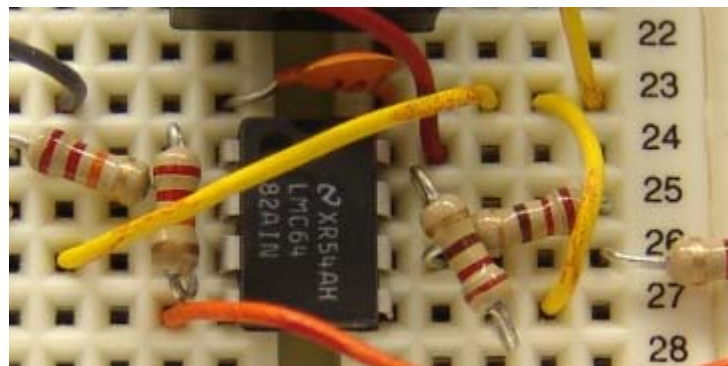
**Figure 19.** Positive terminal of the op-amp is connected from G24 via D20 and A20 to the positive end of the battery.

16. Connect the negative power supply of the op-amp, refer to figure 20.



**Figure 20.** Negative terminal of the op-amp (D27) connected to the negative terminal of the battery via I29 and J29.

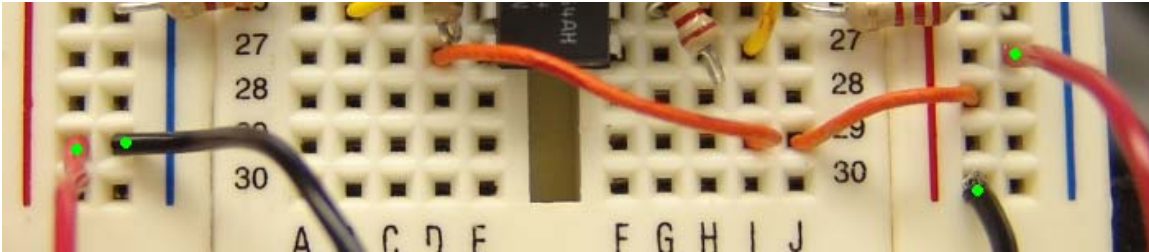
17. The most important connection is to hook up the Chua diode to the rest of the circuit. The connections for doing so are shown in figure 21. I have zoomed in to avoid confusion. The connection is shown by the two yellow wires from 23 to 26 and 27.



**Figure 21.** Connecting the Chua diode.



18. Wire up the batteries as shown in figure 22. **CAUTION: We assume that the red wire is the positive terminal of the battery and the black wire is the negative terminal. Be extremely careful in this step.** You are done!



**Figure 22.** Positive and negative terminals of the battery (green dots).

#### 4. Measurement on PC as an oscilloscope

1. Plug the microphone in the Line-In or Microphone jack of your computer.
2. Select appropriate input (Line-In/ Microphone) from the volume control of your system.
3. Observe the voltage waveform (time series) and/or double scroll strange attractor on the PC using the following software available at [www.chuacircuit.com](http://www.chuacircuit.com)
4. Choose “sound card capture” option in the software. You can also use several other PC oscilloscope softwares available on internet.
5. Vary the potentiometer between the two capacitors and observe the evolution route to chaos and listen to the sound of chaos through the computer’s speaker. An osqoop screenshot is shown in figure 23.

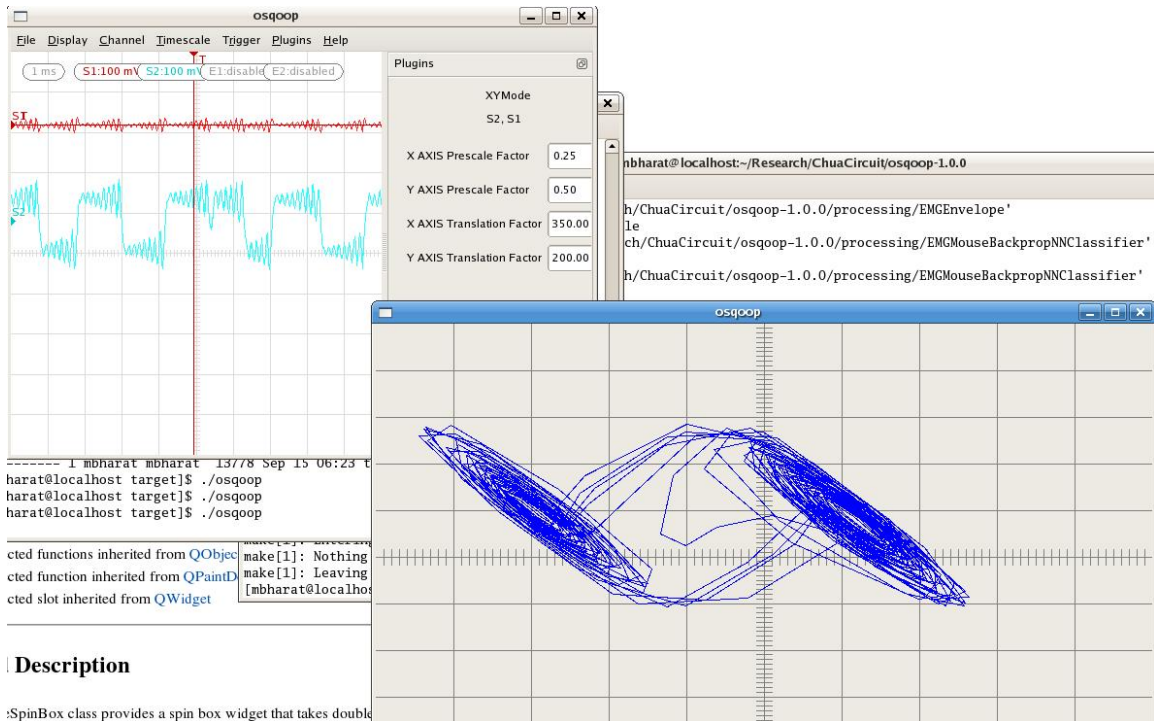
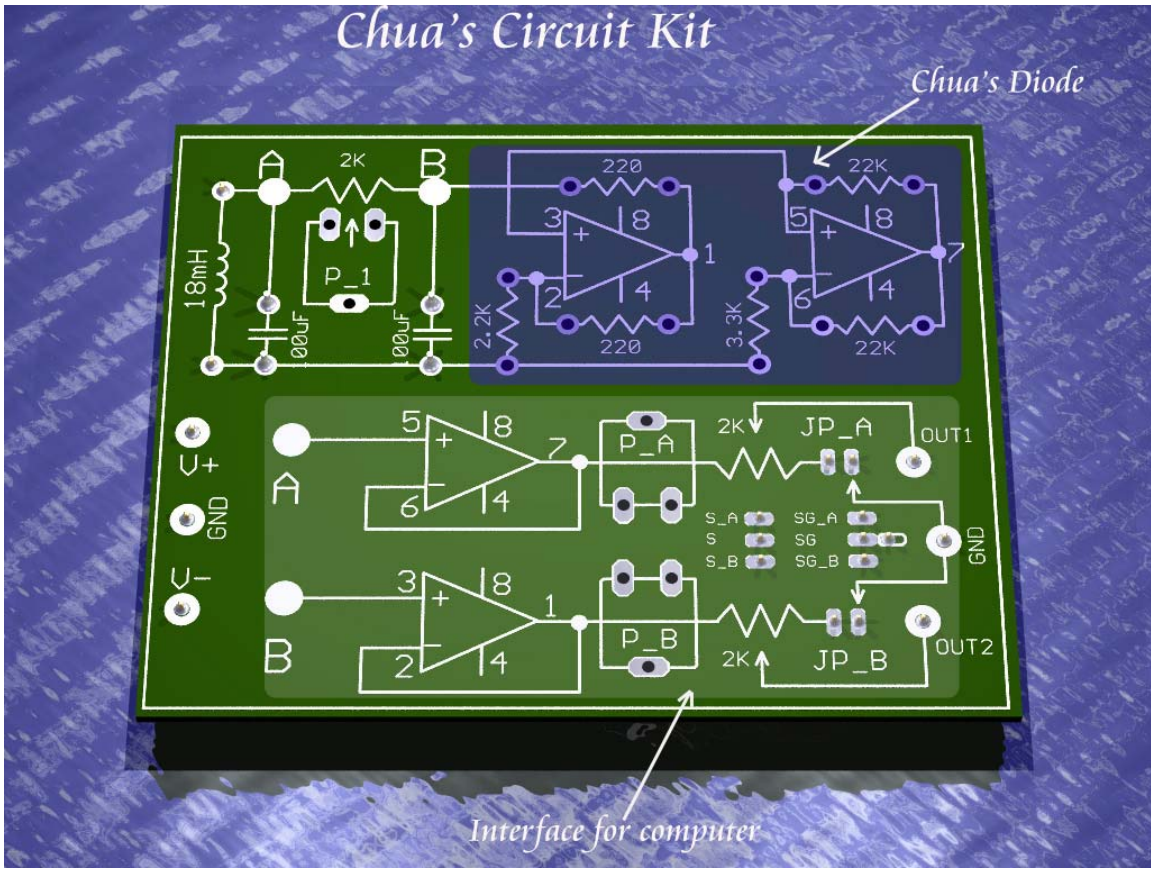


Figure 23. Screenshot of osqoop

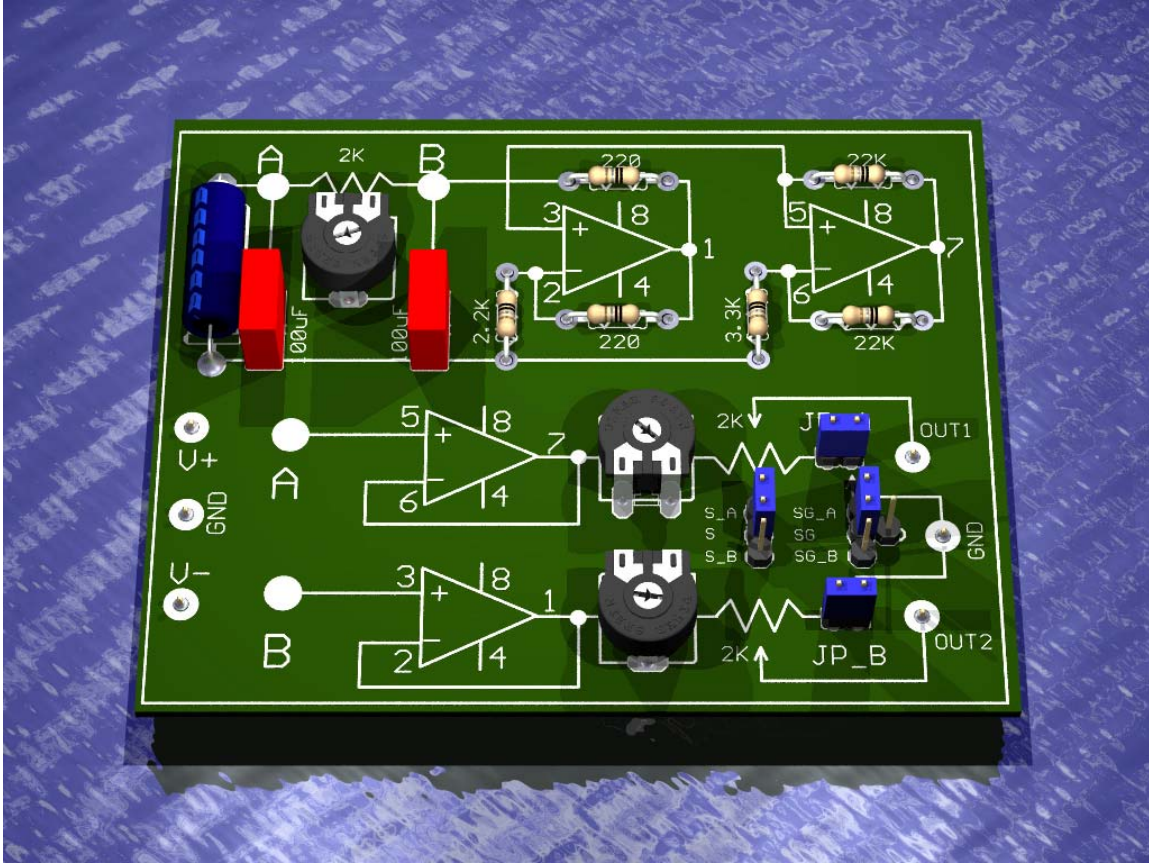
## 5. Chua's Circuit Kit

If you want to build Chua's circuit in just 5 minutes, you can write to us for a pre-designed PnP (plug and play) kit where you just have to plug the components and observed the wave forms on laptop/desktop PC computer. The kit contains the plug and play Printed Circuit Board for Chua's circuit as shown below along with required components that you need to insert into the PCB.



**Figure 24.** Chua's circuit printed circuit board.

The kit will look as follows after you will insert the components to it



**Figure 25.** Completed Chua's circuit kit

You need to buy a pair of nine-volt batteries to power the kit and insert the microphone jack to the laptop.

## 6. FAQs

🔧 Unable to arrange components?

You can request a complimentary complete snap-kit based PnP Chua's circuit kit by writing to us at:

*WRITE PROF. ROSKA'S LAB ADDRESS HERE*

🔧 Will Chua's circuit work for values mentioned in the paper?

A complete set of Monte Carlo simulations was performed to analyze the robustness of the circuit. The complete results will be published in another paper. We have used these values several times and every time it turned out that the system is working without any problem.

■ *Can I operate Chua's circuit at different frequencies?*

Yes. All you need to do is to replace the components by appropriate components. However, it maybe possible that your soundcard doesn't support those frequency values so you may need to observe the results on an oscilloscope.

■ *What else can I do with Chua's circuit voltage waveforms?*

There are lot more things that you can achieve with these waveforms. Chaos and chaotic systems have applications in science, engineering, economics, finance, music etc. You can sample the waveforms into a commercial mathematical package like MATLAB and do signal analysis.

Check out the wikipedia dedicated to Chua's circuit, chaos theory and non-linear dynamics at [www.chuacircuit.com](http://www.chuacircuit.com) and share your experiences, queries and suggestions.

■ *More questions?*

If you have more queries, you can refer/discuss them in FAQ section of ChaosWiki at [www.chuacircuit.com](http://www.chuacircuit.com)

## **7. Conclusion**

A step-by-step guide for high-school students to build Chua's circuit has been proposed. The goal is not only expose students to the exciting field of Chaos, but also help them in basic circuit construction techniques (understanding different passive components, understanding Dual-Inline-Package (DIP) numbering and basic concepts behind a bread-board).

## **DISCLAIMER**

**Authors are not responsible for any damage that may be caused due to the use of material presented in the paper, kits or the software provided.**

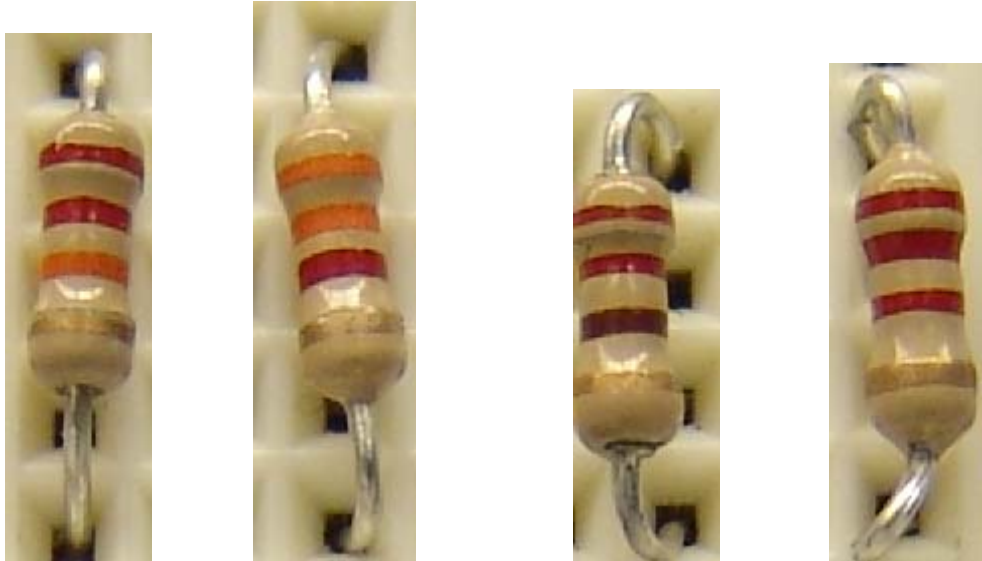
## 8. References

1. [Chua, 1984]. “The Genesis of Chua’s Circuit”. Chua, Leon O. *Archiv fur Elektronik und Uebertragungstechnik*. Vol. 46, No. 4, pp. 250-257, 1992.
2. [Lorenz, 1963]. “Deterministic Nonperiodic flow”. Lorenz, Edward N. *J. Atmos. Sciences*. Vol. 20, pp. 130 – 141. 1963.
3. [Tucker, 2000]. “The Lorenz Attractor Exists”. Tucker, Warwick. *Nature*. Vol. 406, No. 6799. pp. 948-949. 2000.
4. [Chua et. al. 1986]. The double scroll family, Parts I and II. Chua, Leon O., Matsumoto, T. and Komuro, M. *IEEE Trans. On Circuits and Systems*. Vol. CAS-33, No. 11. pp. 1072 – 1200. 1986.
5. [Breadboard Wiki, 2007]. <http://en.wikipedia.org/wiki/Breadboard>. Last accessed: August 15<sup>th</sup> 2007.
6. [Electronics Club, 2007]. <http://www.kpsec.freeuk.com/docs/breadbrd.pdf> Last accessed: August 15<sup>th</sup> 2007
7. [Kennedy, 1992]. “Robust Op-amp Realization of Chua’s Circuit”. Kennedy, Michael P. *Freqz*. Vol. 46, no. 3– 4, pp. 66 – 80, March – April 1992.

## Appendix I: Decoding the Component Values

### Decoding Resistor Values

Resistance values are marked on a resistor by using color codes, refer to figure 26.



Red, Red, Orange:  
22 k $\Omega$

Orange, Orange, Red  
3.3 k $\Omega$

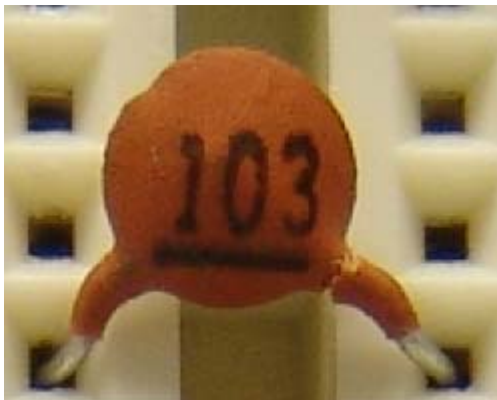
Red, Red, Brown:  
220 ohms

Red, Red, Red:  
.2 k $\Omega$

**Figure 26.** Different resistor values used in Chua's circuit

### Decoding Capacitor Values

Capacitor values are marked on the capacitor. Refer to figure 27.



10 nF



100 nF

**Figure 27.** Capacitor values to be used in Chua's circuit

## Appendix II: Operational Amplifiers

### Op-Amp Numbering

An operational amplifier (op-amp) is an electronic device that can perform a variety of *mathematical (addition, subtraction) operations* and also *amplify* signals. You don't need to understand how an op-amp works, all you need to do is wire it up. Note that in circuit diagrams, op-amps are represented by triangle symbols. The operational amplifier and other integrated chips come in different packaging. The numbering of these pins depends on the type of packaging they are using. The most common type of packaging is DIP (Dual In-line Package) or DIL (Dual In-Line). The numbering method used for these packages is shown in figure 28.

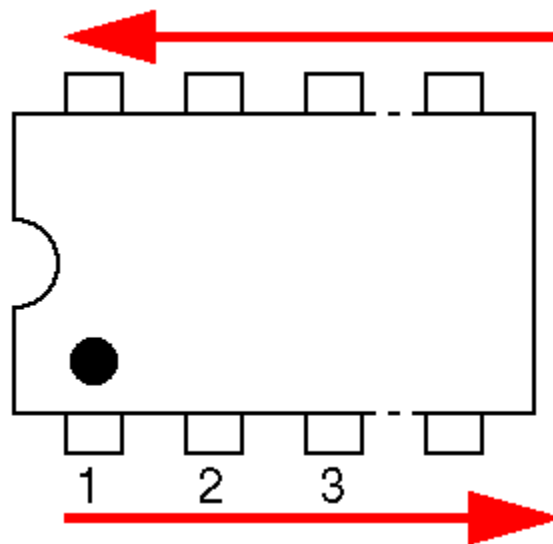


Figure 28. Op-amp DIP package numbering

Always refer to the datasheet of operational amplifier to know the location of positive and negative power supply pins. Most of the datasheets are available at <http://www.datasheetcatalog.com/>.

In the paper, the LMC6482 operational amplifier has been used. However you can also use the TL082 or AD712 op-amps.

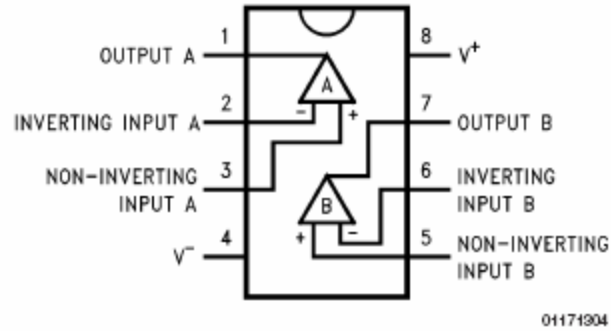
In case you are not using LMC6482 but some other IC ensure the following

- A) Positive power supply pin should replace pin 8 of LMC6482.
- B) Negative power supply pin should replace pin 4 of LMC6482.
- C) Pin 2, 3 and 1 of LMC6482 should be replaced by inverting (represented by minus sign in datasheet), non-inverting (represented by positive sign in the datasheet) and output (the third end of the triangle) of one operational amplifier of the chip.
- D) Pin 6,7 and 8 of TL082 should be replaced by negative, positive and output of another operational amplifier of the chip.

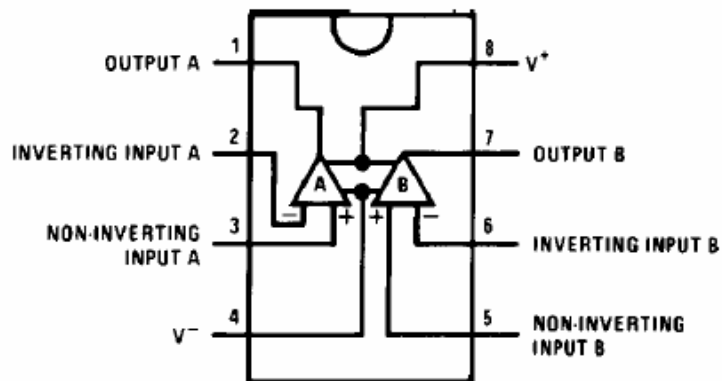


The connection diagrams are shown below.

LMC6482 connection diagram



TL082 connection diagram.



AD712 Connection diagram.

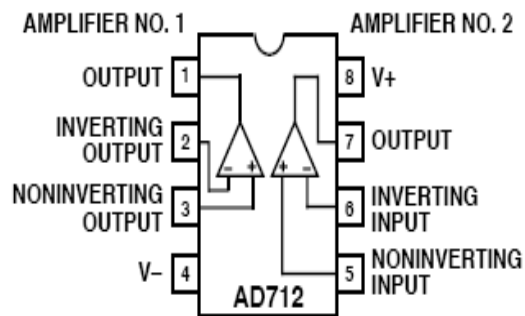


Figure 29. Common Op-Amp pinouts